Platform Design in Targeted Advertising *

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Abstract

Firms compete for slots on an ad-financed platform by paying the platform to display their ads. In its design, the platform decides on its "matching quality" by choosing how much weight to give to each firm's payment while taking into account consumers' preferences. Increasing the quality makes the platform more desirable to the consumers, but also decreases its ad revenue per consumer since it softens competition for exposure among firms. In the optimal design, the platform actively mismatches (matches) the firm with high (low) profit per match by decreasing (increasing) the weight that its payment carries. When consumers' favourite firm enjoys a higher profit per match than its rival, a *neutral* policy in which the platform does not influence matching, improves consumer welfare. The platform is welfare enhancing however, when the consumers' preferred firm earns little per match. Moreover, better information about consumer tastes improves consumer welfare because the platform mismatches the better firm less often.

Keywords: platform design, two-sided markets, advertising contest, digital markets

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1 Introduction

This paper studies the incentives of an advertising-financed platform in its design. Ad-financed platforms earn their revenue by selling limited advertising opportunities to firms who compete for exposure. The two-sided nature of such markets requires the platform to maintain a balance between consumers and advertisers.[1] The platform cares not only about its per-consumer revenue from the advertisers, but also about consumer participation as consumers are valuable to the firms only if they find attention to ads worthwhile. Therefore, it is in the interest of the platform to keep the consumers on board by matching them with those advertisers that deliver a better match value, although such advertiser might not be paying the highest. [2] The element of design that I study in this paper, is the decision of the platform in the relative importance that it puts on the match value for the consumers as opposed to the payments of the firms.

However, there exists a stark trade-off that the platform faces in its design: The implications of matching a consumer with a firm that brings her value are direct for the consumer. But, *better matching can decrease the firms' incentives to pay for exposure.* When a desirable firm, from the consumer's point of view, gets to enjoy better exposure, it does not need to pay the platform as much. In an extreme scenario when the platform cares a lot about the value of a match for consumers, very small payment of a firm with high match value wins considerable exposure compared to other firms. The incentives of a firm that is less valuable for consumers change in the same direction. This is because such firm might get discouraged as now the "design" of the platform does not deliver much "return" for a unit of ad payment. Such incentives, of course, are subject to strategic interactions of firms and the current research aims at investigating those effects in a formal way. As a result, the platform faces a fundamental trade-off between quality of its matching and its advertising revenue.[3] This paper is motivated by the design of such allocation mechanisms and aims at studying how a platform can use its design strategically.

The main question that I address in this paper is what are the implications of the platform's ability to design its allocation mechanism when it faces the trade-off between the match value for the consumers and its ad revenue? What are the implications of strategic behavior of the platform for the advertisers who compete with each other for exposure? Moreover, with regard to the recent policy debates about privacy and use of consumer data on the Internet, an important question is the implications of platform's information

[1] I use the terms firms and advertisers interchangeably

[2] Consider the following excerpt from Facebook addressed at firms who want to advertise:

> " When showing ads, we try to balance two things: (1) Creating value for advertisers by helping them reach and get results from people in their target audience; (2) Providing positive, relevant experiences for people using the Facebook family of apps and services [. . .] Our goal is to match the right ad to the right person at the right time. "

Although what the platforms are claiming to do seems in accordance with the logic of the two-sided markets, the main question that arise is the extent to which they can control the matching mechanism and its consequences. See https://www.facebook.com/business/help/430291176997542 .

[3] See section 1.1 for a review of empirical and theoretical literature on this trade-off.

about consumer preferences. In particular, does better information about consumers improve consumer welfare and if so to what extent?

To answer the above questions, I develop a framework with two firms who compete to earn exposure by paying the platform. Here, I focus on "Display Advertising" which designates a situation where the consumer is not actively searching for a good or service but gets exposed to some unsolicited advertising along her activity on the platform. Prominent examples of display advertising platforms include Facebook, LinkedIn, Instagram, Youtube etc.[4] In the model, consumer earns positive surplus upon matching with either of the firms, nevertheless her match value for one firm is higher than the other. For example, the consumer might prefer seeing an ad for a deal on bicycles by a sports equipment shop in her area of residence rather than a deal for a trip to London by a local travel agency. The firms earn a surplus upon a match as well and this surplus is independent of the match value for the consumers.[5] In particular, it could be the case that the less valuable firm from the consumers point of view, the travel agency in the example above, earns much more per match than its rival. This allows for a situation in which the better firm, from the consumers point of view, has weak incentives for advertising.

The platform (partially) controls the matching technology. The element of design is modelled as the weight that the platform gives to each firm's payment in determining the match probability. The platform offers a higher "matching quality" when it gives more weight to the payment of the "better" firm, here the sports shop, in deciding the exposure it gives to this firm. Giving a higher weight to the better firm therefore indicates a higher quality of the platform, which suits the consumers but it could also affects the incentives of firms in competition for exposure.

Moreover, I study how the platform's information about the consumer preferences, i.e. which firm is the right firm for the consumer, could affect the outcomes. In doing so, I follow a simple approach that the platform could assign a probability on how relevant a firm is to consumers. With regard to the role of data about consumers, one can consider the case that the platform could construct a measure for each firm on how relevant they might be to a consumer, for example after observing browsing history, location, history of purchase etc. Through this parametrization, a simple analysis of how "better information" about consumer taste interacts with outcomes becomes possible.

The model delivers several insights. First, I show how the platform could control the intensity of competition for exposure among sellers through its design. Direct revenue maximization incentives of the platform imply that it actively mismatches (matches) the firm who has a higher (lower) willingness to pay

[4] Another important type of advertising is "Search Advertising" in which the consumer is actively searching for a good or service and the platform matches the advertisers with users by offering them a specific order of results. All the examples above fall in the category of Display advertising except for Google who follows a mixture of Search and Display Advertising. The AdSense business of Google relies on displaying ads to the users through third-party content providers.

[5] This surplus is generated because the consumer might purchase the product, learn about the brand, subscribe to a mailing list that could lead to a purchase in the future etc.

to reach out to the consumers. In other words, the platform makes the "strong" firm handicapped and favours the "weak" firm, in order to intensify the advertising contest among firms. By making the firms more "similar", the platform induces both firms to pay more.

The main result is regarding consumers' welfare and implies that for a large set of parameters of the model, the platform could harm consumer welfare due to its incentives to decrease the quality of matching. More specifically, when consumers' preferred firm enjoys a higher surplus per match, a *neutral* policy, in which regardless of the preferences of the consumers a firm with higher payment could win an ad slot more often, is welfare improving. This follows because a strategic platform optimally dampens the ability of a firm who has stronger incentives to advertise. In the event when firms' incentive to advertise is in line with consumer preferences, i.e when the more desirable firm is willing to advertise more, the platforms optimally chooses to dampen the prominence of the better firm and promote the less desirable firm. However, in the situations where the consumers' less preferred firm is earning a higher revenue per match compared to its rival, the platform could be welfare enhancing.

Another result of the paper is that better information about consumer preferences improves matching quality and consumer welfare. This is due to the effects of better information in inducing the platform to mismatch the better firm less often. When the platform has more accurate information about consumer taste, incorporates such information in its matching mechanism in order to maintain a higher participation of consumers. This will in turn increase the size of the market that the platform offers to the firms and therefore increase its revenue. However, the platform never matches perfectly even with accurate information about consumer taste since with perfect matching its revenue completely dissipates.

1.1 Relation to the literature

This paper is related to the already established literature on information intermediaries (see Lizzeri, 1999; Biglaiser, 1993; Biglaiser and Friedman, 1999, among others). However, most papers in this line of work have been focusing on the incentives for disclosure of information and its welfare consequences. This paper also belongs to the literature of two-sided markets (Caillaud and Jullien, 2001, 2003; Evans and Schmalensee, 2013; Rochet and Tirole, 2006). Most of the studies in this strand of the literature have been focusing on the pricing structure of the platform in different environments (see e.g. Rochet and Tirole, 2006; Armstrong, 2006; Jullien, 2011, for comprehensive reviews).

This paper highlights the trade-off that an intermediary faces between "quality" and its revenue. Several empirical papers have provided evidence on this trade-off. Ghose and Yang (2009); Yang and Ghose (2010); Jerath et al. (2011); Blake et al. (2015) offer evidence that high quality of matching could be a substitute for payment of firms to platform for exposure. Such results are in-line with the statement that when the ads are

highly relevant, and platform's matches the users with relevant results properly, the return on advertising could be small and this might weaken the incentives of the firms to advertise. In the context of ad auctions, Ganuza and Panelva (2010); Hummel and Mcafee (2016); de Cornière and de Nijs (2016); Bourreau et al. (2017) consider the amount of information revealed by the platform to sellers allowing them to target consumers better, which leads to the same trade-off between match quality and platform revenue.

More specifically, this paper contributes to the more recent literature on the topic of platform design and its implications (e.g. Hagiu and Jullien, 2011, 2014; de Cornière and Taylor, 2014; de Cornière, 2016; de Cornière and Taylor, 2019; Inderst and Ottaviani, 2012). In an environment where users incur search cost, Hagiu and Jullien (2011) study the incentives of the platform to divert search. In their model the intermediary has superior information about the match between consumers and firms and generates revenue through fixed payments it receives from firms per consumer visit. Such environment creates an incentive for the platform to introduce noise into the consumers' search process by diverting their search and inducing them to search more. De Cornière (2016) studies search advertising in a setup where firms could target consumers who search for products through the keywords they enter. Although his main focus is on the effects of targeting on prices, he also considers a search engine who can design the matching mechanism (see de Cornière, 2016, Sec 4). In particular, he shows that when the platform can finely *design the matching mechanism*, it is not optimal to match perfectly, even if doing so can be achieved at no cost. However, the mechanisms at play in his work are stemming from the effect of matching quality on consumers' search cost.

The main contribution of the current paper to this line of research is to show that when the platform has market power on the advertising side of the market, even without price competition and consumer search considerations in the buyers side, distortion of matching can take place by platform's ability to adjust the effectiveness of bids through its design.

In another strand of the literature, the intermediary faces the same trade-off of its ad revenue and "quality" but due to the effects on *price* competition among firms (Eliaz and Spiegler, 2011; Burguet et al., 2015; White, 2013; Taylor, 2013; Zhong, 2016). As in all the above-mentioned papers, incentives to trade-off consumer participation for revenue per consumer appear in the current work as well. All these papers (White (2013) being the exception) have consumer search and price competition among firms in their main analysis. My focus, however, revolves around the direct link between firms willingness to pay for exposure and incentives of the platform to use matching quality as a device to extract rent. Therefore, the mechanism through which the platform under-provides (matching) quality is different. Here, the the platform's incentives to influence sellers' competition for exposure through its design is the main driver of the results. Moreover, I am more concerned about how the quality of matching affects the incentives of firms to advertise which has a direct effect of the revenue of the platform. This means the element of design

of platform, through the choice of quality of matching, enters in a stark way.

This paper is also related to the literature on prominence. Armstrong et al. (2009) and Zhou (2011) study the effect of firm prominence on market outcomes in a model of consumer search. However, in those models, prominence of a firm is exogenously imposed whereas in my model, prominence is the result of competition among firms on the platform. In a related work, Armstrong and Zhou (2011) investigate a similar situation. Similar to the current work, they consider firms' competition for prominence through financial payments to the intermediary and analyse the ensuing market outcomes. Similar idea of firms competition for prominence, though through advertising, has been studied by Haan and Moraga-González (2011). This is closer to the current paper as the more salient firm can inhibit the effect of advertising by its rivals. In their paper, though, there is no role for a strategic intermediary and prominence is taking place through an exogenous process. Their main focus is the effects of search costs on advertising, prices and profits.

Among the allocation mechanisms that platforms use, the most common is the Generalized Second Price Auction (see e.g. Varian (2007) and Edelman et al. (2007) for early treatments). Although many treatments are considering ordered auctions (Chen and He, 2011; Gomes, 2014; Athey and Ellison, 2011), I use a contest to represent the platforms' algorithm to allocate ad slots. In doing so, I model competition among the firms for exposure via a contest (see Tullock (1980) and Dixit (1987) as classic examples and Konrad (2007) for a comprehensive review). In a related model where consumers first search for firms whose ads are more salient, Haan and Moraga-González (2011) use a contest to model advertising as well.

This paper is organized as follows. We start by describing the model in section 2. The equilibrium analysis of advertisement of firms is presented in section 3.1. Sections 3.2 and 3.3 consider decisions of consumers and platforms respectively. In section 4.1 I present the main results on the welfare effects and section 4.2 considers the effects of platform's better information about consumer taste. Section 5 concludes.

2 The model

Description of the market and the preferences There are three types of agents in the market: A unit mass of consumers, two firms $i = 1, 2$, and an advertising-financed platform. Consumers are not actively looking for a good or service but the platform can show them unsolicited advertisings of the firms.[6] The consumers are homogeneous in their preferences for the firms. In particular, in state $\theta = 1, 2$ consumers prefer firm $i = \theta$ and derive utility u. When they are matched with firm $i \neq \theta$, they get 0. The common prior is that the two states are equally likely. Each firm i also gets a revenue of r_i, independent of the state,

[6] Hence display advertising.

when it is matched with a consumer. [7]

Information structure Before designing its matching technology, for example using its historical data about the consumers, the platform forms a belief about the state. I use a parameter $\mu \in [0,1]$ as the probability that the platform assigns to the case where $\theta = 1$, i.e the consumer prefers firm 1. Parameter μ summarizes how accurately the platform knows which ad is more relevant to the consumers. [8] Consumers can learn about a firm only through the firm's advertisement on the platform. That is, although the consumers know what kind of product they would like to encounter, they do not know the identity of the firm who offers such product and therefore rely on the ads displayed by the platform. I assume that r_1 and r_2 are known by all the agents. I normalize the outside option of the firms and the consumers to zero.

Platform's technology The platform is strategic and uses its technology to allocate ad slots to firms. Using the platform is free for the consumers, but there is only one ad slot per consumer and firms have to engage in a competitive bidding process, designed by the platform, to win the slot. To allocate the ad slots, the platform uses a matching function in which the firm's payment to the platform is weighted by a *score* that is decided by the platform. The probability that firm i with payment a_i and score q_i is shown to a consumer is [9]

$$S_i(\cdot) = \begin{cases} \dfrac{q_i a_i}{\sum_j^2 q_j a_j} & \text{if } \sum_j q_j a_j \neq 0 \\ 0 & \text{if } \sum_j q_j a_j = 0 \end{cases}, \qquad i = 1,2$$

To simplify the notation, I normalize the scores by q_2, and define $q \equiv \frac{q_1}{q_2}$ and write the matching probabilities in the following equivalent form: [10]

$$S_1(a_1, a_2; q) = \frac{q a_1}{q a_1 + a_2}, \qquad S_2(a_1, a_2; q) = \frac{a_2}{q a_1 + a_2}. \tag{1}$$

[7] Therefore knowing the firms' revenue does not reveal any information about consumer preferences to the platform.

[8] Note that I do not model the information generation process through which the platform reaches a (posterior) belief about the consumer preferences. Instead, I treat μ as a parameter to study the effects of platform's accuracy of information about the consumer preferences on its design and the outcomes.

[9] This functional form could be justified by the method of weighting already practiced by the ad-financed platforms such as Facebook as mentioned in footnote 2. The platforms use the notion of *quality or relevance score* which is used to adjust the advertisers bids. Define the quality score of firm i as a $\hat{u}(u_i)$. Let the probability that firm i is shown to a consumer, when it has an ad expenditure of a_i and a value to consumer u_i, be the following

$$S_i(a_i, u_i; a_{-i}, u_{-i}) = \begin{cases} \dfrac{a_i \hat{u}(u_i)}{\sum_{j=1}^2 a_j \hat{u}(u_j)} & \text{if } \sum_{j=1}^2 a_j \hat{u}(u_j) \neq 0 \\ 0 & \text{if } \sum_{j=1}^2 a_j \hat{u}(u_j) = 0 \end{cases}, \qquad i = 1,2$$

Dividing both the numerator and the denominator of $S_i(\cdot)$ by $\hat{u}(u_2)$ and defining $u \equiv \frac{\hat{u}(u_1)}{\hat{u}(u_2)}$ we get the same form as in eq. (1).

[10] Drawing on the literature of contests (see for example Corchon and Serena, 2016; Jia et al., 2013; Konrad, 2007, for comprehensive reviews), S is called a *contest success function*.

where $q \in [0, +\infty)$ is chosen by the platform and represents the relative weights that the platform gives to firms. Therefore, when $q > 1$, the platform is giving an advantage to firm 1 and when $q < 1$, the platform is favouring firm 2. Observe that for any strictly positive payment by both firms, when $q \to +\infty$, the fraction of consumers matched with firm 1 (firm 2) tends to 1 (to 0) and it tends to 0 (to 1) when $q \to 0$. From the consumers' point of view, q can be seen as the platform's *matching quality* that denotes the advantage that a platform gives to the consumers' favourite match. I use the same term for q hereafter. This is a useful convention because throughout most of the paper I assume that firm 1 is preferred by the consumers which makes the terminology more natural.

Consumer's cost of attention Consumers have the same preferences for the firms but are heterogeneous in their cost of paying attention to ads. Each consumer incurs a cost t if she pays attention to ads and I assume that t is distributed uniformly over the interval $[0, T]$ with $T > 0$. As in Zandt (2004); Anderson and De Palma (2009), this modelling choice aims at capturing the idea that the consumer has limited ability to process or wishes to pay limited attention to the received information (see Renault, 2015, for a comprehensive survey). Alternatively, this cost of attention could also be interpreted as the cost of clicking on an ad by the consumer.

Timing The game unfolds as the following

1. The platform sets its matching quality q
2. Consumers observe whether they prefer firm 1 or 2
3. Firms and consumers observe the platform's choice of q
4. The followings happen simultaneously
    - Firms decide their a_i, i.e. how much to pay the platform per potential consumer
    - Consumer decides whether to pay attention to ads
5. Matching takes place and payoffs are realized

Discussion of assumptions Most of the literature on ad-financed platforms assumes platforms exercise a uniform price on heterogeneous advertisers as in Anderson and Coate (2005). In here I use a matching mechanism which resembles a contest (i.e. a full information all-pay auction) instead for two main reasons: (i) As it is practiced by the platforms and also from the view-point of the firms, a contest would fit better with the description of the market. Such modelling choice is in line with the practice of major ad-financed platforms such as Facebook and LinkedIn as they generally ask firms to set an advertising budget to be spent over a targeted profile of consumers for a specified time period. This is as if all competing firms

pay the platform beforehand and the matching results can be observed *ex-post*. [11] In effect, the firms are participating in several auctions, that is for each ad slot, there is an auction that is run in the background and at the end of the time period a firm might have "competed" in hundreds of auctions. However, from the point of view of the firms, this process is simply paying the platform a sum and letting the platform to match the consumers using its internal processes. (ii) One of the main goals of this paper is to study how the platform could strategically use the notion of scores, i.e. weighting the firms' bids to determine their *effective* matching probability on which the platform has full control. In reality the platform does not commit to a score but upon observing the firms' information (bid, reaction of consumers etc.) decides on it. Here, through a change of timing and information structure, I assume that the platform observes the firms' valuation for a match r_i and then commits to the scores q_i (see footnote 2). Moreover, I have assumed that consumers know the surplus, r_i, that each firm earns upon matching with them as well. This is consistent with the description of the market explained in section 1. Using the example of the deals offered by a sports equipment and travel agency, consumers might know that in general, the travel agencies earn higher margins and therefore are more likely to pay more for advertising compared to the other firm.

Moreover, I assume that firms only compete for exposure and competition in other dimensions, such as in prices, is not modelled here. Although, such generalization could make for a richer model, here I choose to highlight the idea that in display advertising, although firms are competing for prominence, they do not necessarily belong to the same market. [12] Thus, I do not put any specific restriction on the firms' revenues, r_i's, to capture the idea that the firm which is preferred by consumers is not necessarily getting a higher surplus than the other firm at the time of decision for advertising expenditure. This is further discussed in section 4.1.

3 Equilibrium analysis

The game is solved backwards. Assuming that the platform has chosen its matching quality q, I start from the second stage and characterize the equilibrium of the advertising contest. Then, given equilibrium advertisement, I solve for consumers' decision whether to join the the platform. Finally, I move up the game tree and solve for the optimal matching quality of the platform.

[11] See for example the instructions that facebook provides for the advertisers on setting a daily or lifetime *budget* for their ad campaign here https://www.facebook.com/business/help/527780867299597 By setting a budget, firms are paying the platform beforehand, regardless of the mechanism through which their ad is reached to the users. In other words, although other mechanisms such as a Generalized Second-price auction might be at work every time there is an advertising slot to be allocated, firms do not participate in such auction thousands of times and for each of those slots. Rather, they set a budget and leave it to the platform to decide how often and to whom they get matched.

[12] As mentioned as an example in section 1, one could imagine a scenario that two firms are offering *deals* to the consumers and the consumer derives a higher value from one deal compared to the other. Although these firms might belong to the same market and compete in what deals they offer, it is not necessarily so.

3.1 Advertising subgame

The advertising game can be solved in isolation. Given a choice of q by the platform, firms decide on how much to pay per potential match and, simultaneously, consumers decide on whether to participate, i.e pay attention to ads. Focusing on the firms problem, given a measure of participating consumers M, firms engage in a competitive bidding process in which they simultaneously decide the amount they pay the platform per potential consumer. For each firm, $i = 1, 2$, given all payments to the platform, a_i, each consumer is allocated to firm i with probability S_i and to firm $-i$ with probability $S_{-i} = 1 - S_i$. The payoff of firms is then as follows

$$\pi_i(a_i, \cdot) = M\left[S_i(a_i, a_{-i}; q)r_i - a_i\right], \qquad i = 1, 2, \quad -i \neq i$$

The first order conditions for profit maximization (omitting the arguments) writes

$$\frac{\partial S_i}{\partial a_i} r_i - 1 = 0, \qquad i = 1, 2. \tag{2}$$

At the optimum, the marginal benefit accrued to the firm, which is the firm's expected monetary value of spending 1 more Euro on advertising is $\frac{\partial S_i}{\partial a_i} r_i$. Now we can proceed to state the first result, summarized in proposition 1. All omitted proofs are in the appendix.

Proposition 1. *In an equilibrium of the advertising subgame, firms advertise proportional to their revenue per potential consumer, i.e*

$$\frac{a_i^*}{a_j^*} = \frac{r_i}{r_j}, \qquad i, j = 1, 2 \quad , j \neq i$$

Proposition 1 implies that the firm with higher surplus per ad, pays more to the platform. This result is intuitive as the firm with higher "return" per Euro tends to "invest" more. We can also explicitly solve for the equilibrium level of firms' payments to the platform per potential consumer:

$$a_i^* = \frac{q\gamma}{(1 + q\gamma)^2} r_i, \qquad i = 1, 2. \tag{3}$$

Where $\gamma \equiv \frac{r_1}{r_2}$ is the ratio of the firms' revenue. Comparing the matching probability of firms at the equilibrium of the advertising game given a choice of q, the share of the consumers allocated to each firm at the equilibrium of the subgame is as follows

$$S_1(a_1^*, a_2^*; q) = \frac{q\gamma}{q\gamma + 1}, \qquad S_2(a_1^*, a_2^*; q) = \frac{1}{q\gamma + 1} \tag{4}$$

Lemma 1. *At the equilibrium of the advertising subgame, for a ratio of firms' revenue per consumer, γ,*

when $q > 1/\gamma$ (respectively $<$) then $S_1 > S_2$ (respectively $<$). At $q = 1/\gamma$ both firms are equally likely to be matched with a consumer.

Proof. Comparing the two expressions in eq. (4) delivers the result directly. □

Lemma 1 indicates the degree of control the platform can exercise through its choice of quality. In other words, through its choice of q, the platform can shift the demand between the firms.

Analysis of the advertising subgame To see how the design of the platform influence firms' competition, we can totally differentiate the firms' first order condition of profit maximization to get

$$\frac{\partial^2 S_i}{\partial a_i \partial q} dq + \frac{\partial^2 S_i}{\partial a_i^2} da_i + \frac{\partial^2 S_i}{\partial a_i \partial a_j} da_j = 0, \quad i, j = 1, 2, \quad j \neq i \tag{5}$$

From eq. (5), we can see how the behavior of the sellers in terms of their reaction to the platform's quality and to each other depends on the shape of the matching function S. The first term in eq. (5) is the effect of the change in quality of the platform on the marginal return on advertising, $\partial S_i/\partial a_i$ of firm i. The second term is the direct effect on how firm i's own advertising could affect its return on advertising, $\partial S_i/\partial a_i$. And the third term is the effect of an increase in the advertising spending of firm $j \neq i$ on the returns on the advertising of firm i. The shape of the matching function, S_i, then becomes important as a concave S_i means that an increase in advertising spending of firm i will decrease the returns earned due to this marginal increase. This speaks to the implications of the platform design on the competition of sellers for prominence. In particular, as I will show below, the platform can exercise its control over the behavior of the firms through its decision on the matching quality, q. Another aspect of the choice of design is the inhibition effect of advertising of one firm on the prominence of other firms.

It is useful to discuss how firms change their ad spending in the equilibrium of the ad contest in response to a change in the quality. To do so we need to consider the direct effect of a change in the quality of the platform (shift in best-reply functions of the firms), as well as the strategic reaction of firms to each other (changes in the slope of the best-replies).

Denote by $\hat{a}_i(a_j)$, the bes reply of firm i to an ad expenditure of a_j by firm j. Total differentiation of firm i's first order condition of profit maximization and setting $dq = 0$ gives

$$\frac{d\hat{a}_i(a_j)}{da_j} \mid_{dq=0} = - \frac{\partial^2 S_i}{\partial a_i \partial a_j} \Big/ \frac{\partial^2 S_i}{\partial a_i^2} \tag{6}$$

Consider the following result. All omitted proofs are in the appendix.

Lemma 2. *For a given ratio of firms revenues, γ, in the neighbourhood of the equilibrium of the ad contest and keeping the quality constant,*

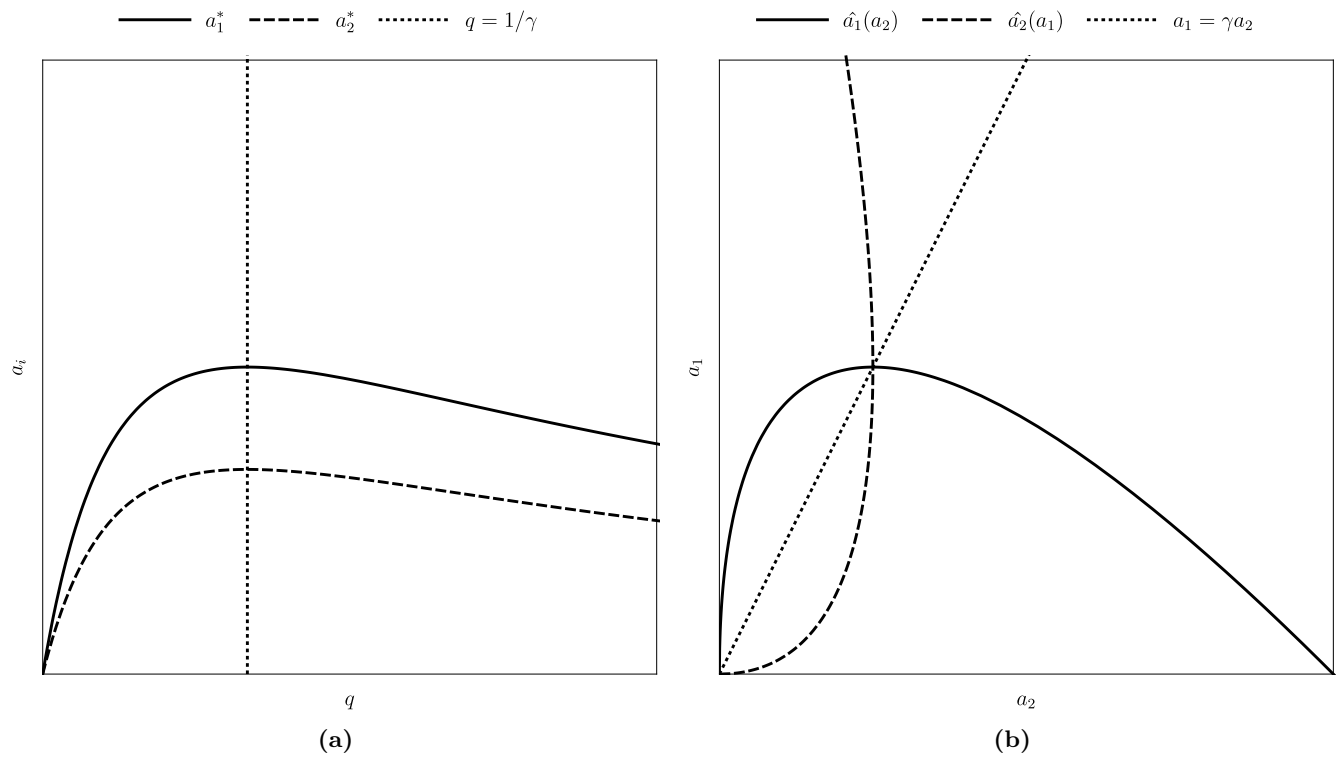

Figure 1: Advertising contest is most intense when $q = 1/\gamma$. Assuming $\gamma > 1$: **(a)** Equilibrium ad payment of firms, a_i^* for given values of quality. **(b)** Best-reply functions of the firms at $q = 1/\gamma$.

I. When $q > 1/\gamma$, $\frac{d\hat{a}_1(a_2)}{da_2} > 0$ *and* $\frac{d\hat{a}_2(a_1)}{da_1} < 0$.

II. When $q < 1/\gamma$, $\frac{d\hat{a}_1(a_2)}{da_2} < 0$ *and* $\frac{d\hat{a}_2(a_1)}{da_1} > 0$.

Similarly, we look at the effect of a change in the quality of the platform on the best-reply functions of the firms while isolating away their strategic reaction to each other. Using the equilibrium condition in eq. (2), we change q while keeping the reaction of firm j constant. This gives

$$\frac{da_i}{dq}\Big|_{da_j=0} = -\frac{\partial^2 S_i}{\partial a_i \partial q} \Big/ \frac{\partial^2 S_i}{\partial a_i^2}$$

Lemma 3. *For a given* γ,

I. When $q > 1/\gamma$, *a decrease in* q *shifts the best-reply function of both firms up*

II. When $q < 1/\gamma$, *an increase in* q *shifts the best-reply function of both firms up*

The above analysis shows that the advertising contest is the most intense when the platform has chosen a level of quality that makes both firms equally likely to win over a consumer. When q is close to $1/\gamma$, the optimal advertising spending of the firms increases as S_i gets closer to $S_j = 1 - S_i$. At $q = 1/\gamma$, we get an equilibrium in which both firms are paying the highest possible amount to the platform. Figure 1 depicts the above analysis. This has implications for the platform decision in setting its matching quality as the platform wishes to induce higher ad payments by the firm. We address this point in detail in section 3.3.

3.2 Consumer participation

Now we proceed to the next step and consider consumers' decision to whether ignore the ads or pay attention to them. Depending on the state , $\theta = 1, 2$, a consumer of "type" θ compares her expected payoff, which I denote by V_θ, with her cost of attention t and pays attention to ads ("participates") only if $V_\theta - t \geq 0$. Mass of such consumers, which we already defined as $M_\theta \equiv \Pr(V_\theta > t) = F(V_\theta) = V_\theta/T$.

When deciding whether to pay any attention to the ads on the platform, a consumer considers how likely it is to see the advertisement of a valuable firm given the quality chosen by the platform and anticipating the advertising spending of the firms. Consumer's expected payoff from paying attention to the ads on the platform is $V_{\theta=1} = S_1(a_1^*, a_2^*)u$ and $V_{\theta=2} = S_2(a_1^*, a_2^*)u$. Substituting for the value of a_i^* from eq. (3), we get the following expression for the expected payoff of a consumer

$$V_{\theta=1} = u\frac{\gamma q}{1+\gamma q}, \quad \text{and} \quad V_{\theta=2} = u\frac{1}{1+\gamma q}, \tag{7}$$

Consider the state of the world where consumers prefer firm 1, that is $\theta = 1$. Then, with a higher q, S_1 increases and therefore consumer can gain more since now she will be more likely to get matched with firm 1. In contrast, in state $\theta = 2$, consumer prefers matching with firm 2 and a higher q *decreases* such probability. A change in q induces a response by firms as well. Differentiating consumer's expected payoff V_θ with respect to q, we can see the effects of a change in matching quality:

$$\frac{dV_{\theta=i}}{dq} = u\left(\frac{\partial S_i}{\partial q} + \frac{\partial S_i}{\partial a_1}\frac{da_1(q)}{dq} + \frac{\partial S_i}{\partial a_2}\frac{da_2(q)}{dq}\right).$$

Using the first order condition of firms in eq. (2) in the advertising subgame and the identity $S_1 = 1 - S_2$ we can see how such a change affects consumers' surplus at the equilibrium of the advertising game

$$\frac{dV_{\theta=i}}{dq} = u\left(\underbrace{\frac{\partial S_i}{\partial q}}_{\text{match quality effect}} + \underbrace{\frac{1}{r_1}\frac{da_1^*(q)}{dq} - \frac{1}{r_2}\frac{da_2^*(q)}{dq}}_{\text{Advertising effect}}\right) \tag{8}$$

An increase in consumer payoff happens only if eq. (8) is positive. The first term, $\partial S_i/\partial q$, is the *quality effect*, i.e. the direct effect of a change in the matching quality of the platform. From eq. (7), this is positive if consumers prefers firm 1 and is negative if they prefer firm 2.

The second term takes into account the strategic response of firms due to a change in q, i.e. *advertising effect*. Recall from proposition 1 that the equilibrium levels of ad expenditures are proportional to the firms revenues. Then it is easy to see that $(1/r_i)(da_i^*/dq) = (1/r_j)(da_j^*/dq)$ which renders the advertising effect

in eq. (8) nil. [13] Thus, the strategic interaction of firms in response to a change in quality does not change the consumer's expected payoff.

Proposition 2. *At the equilibrium of the advertising subgame, consumers' expected payoff $V_\theta(\cdot)$,*

I. Increases in the the revenue of the consumers' favourite firm

II. Increases in platform's bias for the consumers' favourite firm

Since firms advertise proportionally to their revenue, a higher revenue per match by the consumers' favourite firm leads to a higher intensity of advertising by such firm. Therefore, for given value of quality, a higher $r_{i=\theta}$ means higher probability of matching for firm $i = \theta$.

3.3 Platform's decision

In this section we determine the optimal choice of q by the platform. For convenience define the platform's ad revenue per potential consumer from all firms as $A(q) \equiv \sum_i a_i^*(q)$. The platform has partial information about consumers' preferences at the time of making decision about q. Therefore it calculates the measure of participating consumers in expectation. In the context of Internet platforms, this could be a interpreted as the probability of *clickthrough* or *conversion* by the consumer. Denote by $\hat{M}$ the expected measure of participating consumers from the platform perspective, then we have

$$\hat{M} \equiv \mathbf{E}_\theta\{M_\theta\} = \mathbf{E}_\theta\{V_\theta \ / \ T\} = [\mu V_{\theta=1} + (1-\mu)V_{\theta=2}] \ / \ T = u[\mu S_{i=1} + (1-\mu)S_{i=2}]/T.$$

The expected revenue of the platform depends on how much the firms pay per potential consumer, A, and the expected mass of consumers, $\hat{M}$ who pay attention to the ads. Then the platform's payoff writes

$$\Pi(q) = A(q) \ \hat{M}(q, \mu)$$

Maximizing its profit by choosing q, the first order condition for optimal q is

$$\frac{\partial \Pi(q,\cdot)}{\partial q} = \frac{\partial \hat{M}(q,\cdot)}{\partial q} A(q) + \frac{\partial A(q)}{\partial q} \hat{M}(q,\cdot) = 0 \tag{9}$$

This represents the basic trade-off that the platform is facing. Assume that using its past information, the platform puts a higher likelihood on firm 1 to be the more relevant to the consumers, i.e $\mu > 1/2$. Then for a small increase in q, the first term represents the expected gain of the platform in advertising revenue

[13] Although this outcome is not general and is a result of the choice of the matching technology, it makes the model tractable. Generally, the class of all scoring functions in the form of $h_i = q_i a_i$ where q_i is the weight given to the payment of the firm a_i deliver the same result as in here. In the literature of contest models, these are contests with *bias* but without *headstart* (see Kirkegaard, 2012; Li and Yu, 2012; Drugov and Ryvkin, 2017, for examples).

on a marginal increase in the number of consumers who participate. The second term shows the marginal decrease in ad revenue, due to the advantage that firm 1 gets, over the fraction of consumers who are already participating. The same trade-off will be in place in case of $\mu < 1/2$ but for a decrease in q.

Substituting M and rearranging we can write eq. (9) in the following elasticity form (omitting the arguments) which again shows how the platform trades off the expected participation of the consumer with its ad revenue.

$$\frac{\partial \hat{M}/\partial q}{\hat{M}} + \frac{\partial A/\partial q}{A} = 0. \tag{10}$$

Using eq. (10) we can characterize the optimal matching quality of the platform, denoted by q^*. Proposition 3 below presents the main equilibrium result.

Proposition 3. *The optimal matching quality of the platform, q^*, is characterized by*

$$q^* = \frac{1}{\gamma}\phi(\mu)$$

where, $\phi(\cdot)$ is an increasing function of μ and $\phi(1/2) = 1$.

4 Results

Consider the following result that obtains immediately from proposition 3.

Corollary 1. *The platform actively mismatches (matches) the firm with higher (lower) revenue per match.*

Proof. Recall that q was the "relative" weight for the payment of firm 1. A direct corollary of the result in proposition 4 since q^* inversely related to the revenue ratio γ. □

The interpretation of the result in proposition 3 and corollary 1 is the following: From proposition 1 and the analysis in section 3.1, we know that firms bid proportional to their revenue per match and that the platform wants to induce both firms to advertise more by creating a bias in favour of (against) the firm with lower (higher) revenue per match. When γ is small, firm 1 has lower incentives to advertise and firm 2 can gain prominence with a smaller bid. A higher (lower) q, therefore encourages firm 1 (firm 2) to bid higher and at the same time keeps firm 2 (1) on its toes.

Proposition 3 is reminiscent of models of *handicap auctions* and *biased contests* (see Clark and Riis, 2000; Eso and Szentes, 2007; Franke, 2012; Kirkegaard, 2013, for example). In these models, the designer wishes to favour the *weaker* opponent and handicaps the *stronger* player in order to illicit higher bids.

4.1 Does a strategic platform improve consumer welfare?

One of the main motivations of this paper is to understand how the ability of a platform to design its technology can affect the outcomes in equilibrium. Throughout the model we have seen that two main forces drive the outcomes: (i) Incentives of the individual firms in competition for exposure; and (ii) Incentives of the platform for maximizing profit. Proposition 3 suggests that the platform intensifies the competition between firms by creating a bias in such a way that counteracts the effects of firms' payments for exposure. An important question here is: Does the platform gives better prospects in matching with the consumers' more desirable firm?

In order to better understand the role of the platform, it is useful to separate these two main effects. To do so, I consider a simple benchmark where firms compete for exposure through a non-strategic platform that cannot influence the matching using its only instrument q. In other words, consider a case where the platform does not influence the probability of matching and the only determinant of the match probability of each firm is the amount they spend on advertising through the platform. That amounts to a *neutral* policy in which the platform sets a $q = 1$. By setting q to 1, when firms spend equal amounts, the matching would be random. Therefore, under this neutral policy, the platform does not influence the weight that each firm's bid carries. Such *neutral* policy implies that the firm who bids higher to capture a proportionally larger share of the participating consumers.

In the following, I compare the outcome of advertising contest in the equilibrium and under the neutral policy. To simplify the exposition, assume that consumers prefer firm 1 to 2 and therefore a larger q implies better prospects of matching with firm 1. This also implies that consumer surplus will be higher as suggested by part II of proposition 2. The main result states that,

Proposition 4. *For a given μ, compared to a neutral policy, when the consumers' preferred firm enjoys a high enough (low enough) revenue advantage over its rival, the platform harms (improves) consumer welfare.*

Proof. With a neutral policy, we have $q = 1$. In the equilibrium, platform sets $q^* = \frac{1}{\gamma}\phi(\mu)$. When consumers prefer firm 1 to 2, compared to the neutral platform, consumers worse off in equilibrium if $q^* < 1$. This happens when $\frac{1}{\gamma}\phi(\mu) < 1$ which gives $\gamma > \phi(\mu)$. Therefore when consumers prefer firm 1 and $r_1 > \phi(\mu)r_2$, a neutral policy improves consumer surplus. Similarly, under the assumption that consumers prefer firm 2, the platform harms consumer welfare in equilibrium if $q^* > 1$. Then if the revenue advantage of firm 2 is large enough, that is when $\gamma < \phi(\mu)$, consumers better off under neutral policy. □

Section 4.1 depicts the range of parameters for which a *neutral* policy provides higher consumer surplus than equilibrium.

The neutral policy implies that the firm who bids higher captures a proportionally larger share of the

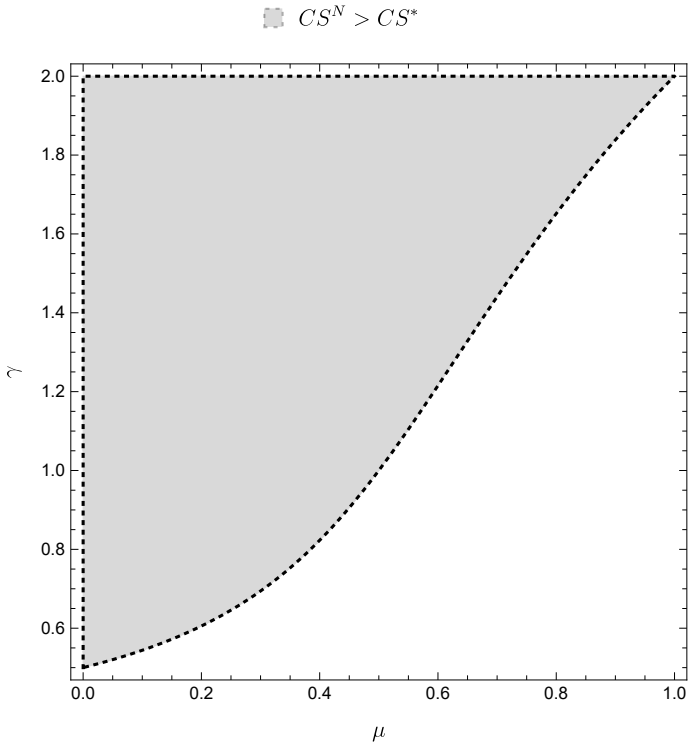

Figure 2: Consumer surplus in equilibrium and under the neutral policy
The shaded area depicts range of parameters for which consumer surplus under the neutral policy CS^N is larger than consumer surplus in the equilibrium CS^*

demand (participating consumers). Therefore, in the event that the better firm, from the consumer point of view, is earning more, a neutral policy allows this firm to get matched more often. On the other hand, following the result in proposition 3 and corollary 1 , the strategic platform gives a disadvantage to the "strong" firm and favours the "weak" firm in order to illicit higher bids. If incentives of the firms are in accordance to the preferences of the users, i.e. the ideal firm from the consumers point of view also happen to be the firm with higher revenue per match, a neutral policy could lead to better outcomes than equilibrium.

Given our assumption that firm 1 is the preferred firm by the consumers, the platform can improve consumer welfare but only in the cases where firm 1 is earning less per match compared to firm 2. This means that the intermediary plays a positive role in situations where a firm that is more valuable to the consumers does not have enough mark-up per consumer to advertise. This result is also in line with the public and political economics literature in the analysis of affirmative action, levelling the playing field and positive discrimination (e.g. Epstein et al., 2011; Franke, 2012) . Here, the platform is favouring the weaker player for two main reasons: The first is the direct private revenue maximizing incentive because of which the platform wants to handicap the stronger player and favour the weak player; Second, this is reinforced as not favouring firm 1 which is more valuable to the consumers, also hurts consumer participation which indirectly affects the revenue of the platform.

This result is also related to Hagiu and Jullien (2011). In a setup with consumer search and fixed payment of firms per match, they show that the platform has the incentive to divert consumers away from

their favourite firm because the platform receives a (fixed) per-click or per-purchase commission. Although a similar logic follows in my model, the mechanism at play is different. In Hagiu and Jullien (2011), the platform decreases the matching quality (diverts) in order to induce *consumers* to search more and therefore increases its revenue through more clicks. Whereas here, the platform decreases its matching quality in order to induce the *firms* to bid higher. And depending on which firm earns more, the effect could go in either direction.

Platform never matches perfectly Following the convention that consumers prefer firm 1, a perfect level of quality by the platform implies $q \to +\infty$ which means the platform steers consumers to firm 1 regardless of firms' level of advertising.

Proposition 5. *Perfect quality is never an equilibrium even when the technology of the platform is costless.*

Proof. Using the expression for equilibrium advertising of firms in eq. (3) we can see that $\lim_{q\to+\infty} \frac{q\gamma}{(1+q\gamma)^2} r_i = 0$ for $i = 1, 2$. As a result $\lim_{q\to+\infty} A = 0$ which renders the profit of the platform nil as V is bounded above by u_1. □

The result in proposition 5 comes from the fact that with perfect quality none of the firms have any incentives to advertise, leaving the platform with zero profit. Therefore, under private profit maximizing incentives the platform will never perfectly match firm 1 with the consumers.

4.2 Platform's information about consumer preferences

One of the main goals of this paper is to study how the platforms' knowledge about consumer preferences affects the outcomes. In the previous section, we characterized the optimal matching quality of the platform, q^* under the assumption that the platform has some information about consumers' taste summarized in a single parameter μ. In this section, I study the effects of more accurate information of the platform about the true preferences of the consumers.

More accurate information about preferences improves matching Using the results in section 3.3, we can study how accuracy of the platform's information about consumer preferences affects the outcomes. Note that again for the ease of exposition I consider the case where the consumers prefer firm 1 to 2 and therefore a higher μ indicates more precise information about the state by the platform.

Corollary 2. *The optimal matching quality of the platform q^*, increases with the accuracy of the platform's information about consumer preferences, μ.*

Proof. Proof is directly following the result in proposition 3 that $\phi(\mu)$ is an increasing function. □

The intuition for this result is the following: Given a fixed probability of consumer participation, the platform always has the incentives to maximize its revenue through granting the same level of exposure to both firms, i.e. to make them more similar. However, knowledge about the preferences of the consumer allows the platform to bias its matching in favour of the consumers' preferred firm in order to avoid missing out on the potential revenue due to a larger market size M. Figure 3 depicts how the optimal q changes when the platform goes from a situation of $\mu = 1/2$ to $\mu = 1$. Platform's information about consumer taste then could improve the welfare result as the platform would ensure improving the matching of the consumer with their preferred firm more often.

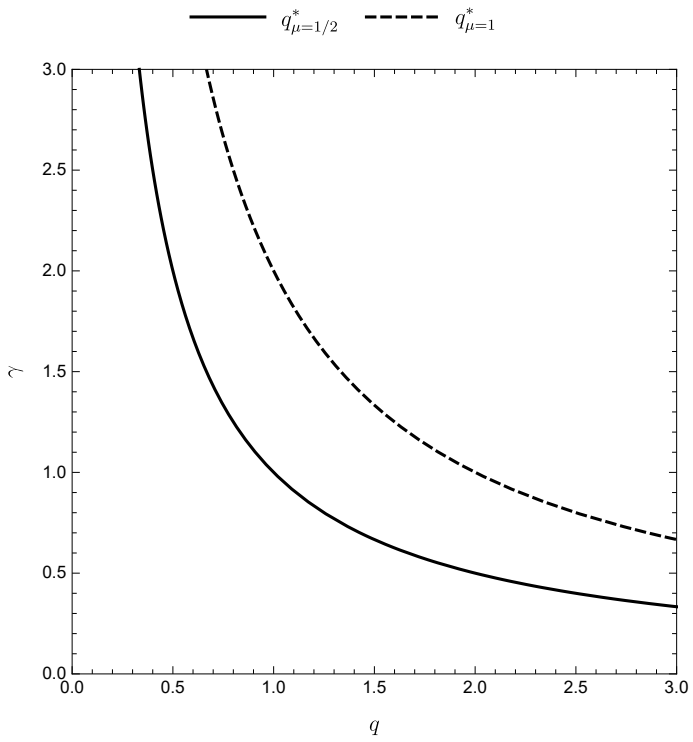

Figure 3: Provision of quality by the platform as a function of information about consumer preferences μ
Assuming consumers prefer firm 1 , provision of quality by the platform for the case when it has no information about the preferences of the consumers $\mu = 1/2$ and the case when it perfectly knows the consumer taste $\mu = 1$.

Which firm becomes prominent more often? Assuming consumers prefer firm 1, a relevant question here is whether in equilibrium, the platform promotes firm 1 more than firm 2. Specifically, what is the behavior of the platform when the less preferred firm has a high mark-up and therefore higher incentives to advertise compared to its rival. Recall that when $q < 1/\gamma$, firm 2 becomes prominent more often, that is $S_2 > S_1$ and consider the following result:

Corollary 3. *When $\mu > 1/2$ (respectively $\mu < 1/2$), then in equilibrium firm 1 (firm 2) is getting more exposure regardless of γ.*

Proof. From lemma 3 we know that advertising spending of the firms decreases when the platform decreases its quality below $1/\gamma$. Therefore, the platform never chooses quality below $1/\gamma$. That means keeping

participation constant, the platform loses revenue by setting quality below $q = \frac{1}{\gamma}$. Moreover, from item II consumer's expected payoff from joining the platform is increasing in μ and therefore decreasing μ hurts participation which decreases platform's revenue as well. Therefore, any choice of $q < 1/\gamma$ is dominated. □

Corollary 3 has implications for the role of the intermediary in the prominence of firm 1. It suggests that for any level of revenue ratio of the firms, γ, when firm 1 is preferred by the consumers, *and the platform knows this*, the platform ensures that such firm becomes prominent more often, that is $S_1^* > S_2^*$. The implication is that even under private profit-maximizing incentives, the equilibrium behavior of an omniscient platform with perfect information about consumers' and firms could lead to a desirable outcome, i.e. the consumers' preferred firm becomes more prominent than its less valuable rival.

5 Concluding remarks

In this paper I studied the strategic behavior of an ad-financed platform in the context of display advertising. An ad-financed platform allows firms to reach out to consumers who are known to have positive demand for both products but prefer one firm to the other. The platform induces a competitive bidding contest among firms and controls the intensity of competition among firms by setting the quality of matching between firms and consumers.

In its decision on design, the platform faces a trade-off between consumer participation and ad revenue. As the platform improves the quality, the advertisers will have less incentives to advertise as this softens their competition for exposure. From an ad revenue point of view, therefore, the platform prefers to decrease the quality. However, by decreasing quality, the platform will also harm consumers as now they find the high quality firm less often which discourages consumer participation. As a result, the private profit-maximization incentives of the platform could distort matching. The main implication of such behavior by the platform is that it could harm consumer welfare, compared to a neutral policy in which the platform does not influence the matching through its design. In particular, in the event that the better firm also earns a higher surplus per match, the platform dampens its exposure to capture a part of the surplus. However, in the event that the consumers' favourite firm does not earn much as its rival per match, the platform could benefit the consumers.

This paper also studies the implications of platform's information about consumer preferences. I showed that the matching quality of the platform could increase when the platform has more accurate information about consumers taste for firms. Although through its design, the platform chooses its quality in such a way to intensify the ad contest among firms, with more accurate information about consumer preferences it will always give more exposure to consumers' preferred firm even when such firm has lower mark-up and

therefore lower incentives to advertise. This is the consequence of the platform's incentive to dampen the prominence of the firm with higher incentive to advertise (due to higher mark-up) being offset by a loss in consumer participation. This implies a positive role of the platform as it ensures a larger part of the demand will always go to the consumers' favourite firm.

The above result suggests that the private profit-maximization incentives of the platform could harm consumer welfare in situations where the consumers' ideal firm does not have a high mark-up per match. This result also indicates how the structure of the product market and the platform behavior could interplay. This interplay of product design and platform design has been recently studied by de Cornière and Taylor (2019). Departing from the framework of Armstrong and Vickers (2001), they consider firms who compete in the utility space and show how *congruence* or *conflict* between firms' mark-up and consumer's utility could interplay with an inherently biased platform. They show that whether firms and consumers have *congruent* or *conflicting* payoffs has strong implications for the effect of a bias imposed by an intermediary. In the current paper, although bias is emerged endogenously, the effects of the bias still depend on whether the firm that delivers a higher utility to the consumer is earning more than its rival or less. Using the terminology of de Cornière and Taylor (2019), congruence or conflict becomes important in both market.

The results point to the importance of the structure of the product markets and the ratio of consumer and producer surplus therein. Since the platform considers the incentive of the firms to advertise as well as the intensity of consumer preferences for the product in its design, an interesting question is whether such global and widespread influence of the platforms could interact with the structure of the markets and possibly influence them. For example, would the design of the platform induce firms to change their product design? Would the design of the platform induce firms to only offer products with higher mark-up so that they could compete with their rivals in contest for prominence? These questions are left for the future.

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APPENDIX

For convenience in the proofs of proposition 1, lemma 2 and lemma 3 define $h_1(a,q) = qa_1$ and $h_2(a,q) = a_2$, we have $S_i = \frac{h_i}{h_i+h_j}$.

Proof of proposition 1

Proof. Consider the first order condition of profit maximization by firms in eq. (2). Then we can write

$$\frac{\partial S_i}{\partial a_i} = \frac{\partial h_i/\partial a_i}{h_i} S_i(1-S_i)$$

Plugging in the above and using the identity $S_i = 1 - S_{-i}$ we can summarize both equations into one and get $\frac{\partial h_i/\partial a_i}{h_i} S_i S_j r_i = \frac{\partial h_j/\partial a_j}{h_j} S_j S_i r_j$. For the scoring functions h_i used in eq. (1), we have $\frac{\partial h_i/\partial a_i}{h_i} = \frac{1}{a_i}$ for $i = 1, 2$ from which the result follows immediately. □

Proof of lemma 2

Proof. We need to sign eq. (6). The denominator is always negative:

$$\begin{aligned} \frac{\partial^2 S_i}{\partial a_i^2} &= \frac{\partial}{\partial a_i}\{\frac{\partial h_i/\partial a_i}{h_i} S_i(1-S_i)\} \\ &= \left[\frac{\partial^2 h_i/\partial a_i^2}{h_i} S_i(1-S_i) - \left(\frac{\partial h_i/\partial a_i}{h_i}\right)^2 (2S_i)\right] S_i(1-S_i) < 0 \end{aligned} \tag{11}$$

Note that the above result relies on $\frac{\partial^2 h_i}{\partial a_i^2} \leq 0$ for $i = 1, 2$. Which holds as $h_1 = a_1 q$ and $h_2 = a_2$.

Now we sign the numerator.

$$\begin{aligned} \frac{\partial^2 S_i}{\partial a_i \partial a_j} &= \frac{\partial}{\partial a_j}\left\{\frac{\partial h_i/\partial a_i}{h_i} S_i(1-S_i)\right\} \\ &= \frac{\partial h_i/\partial a_i}{h_i} \frac{\partial S_i}{\partial a_j}(1-2S_i) \end{aligned}$$

Note that $\partial S_i/\partial a_j < 0$. However, the sign of the expression $(1-2S_i)$ depends on whether the firm has a matching probability higher than its rival. When firm i becomes prominent we will have the case that $S_i > S_j$ which implies $1 - 2S_i < 0$. Recall from lemma 1 that $q > 1/\gamma$ implies $S_1 > S_2$ which means $1 - 2S_i < 0$. Thus,

$$\frac{\partial^2 S_1}{\partial a_1 \partial a_2} = -\frac{\partial^2 S_2}{\partial a_2 \partial a_1} > 0.$$

Therefore the signs of the cross-partials are the opposite although their absolute effect is equal. □

Proof of lemma 3

Proof. Since from eq. (11) S_i is concave in a_i, $\frac{\partial^2 S_i}{\partial a_i^2}$ is always negative. It remains signing the numerator

$$\begin{aligned}\frac{\partial^2 S_i}{\partial a_i \partial q} &= \frac{\partial}{\partial q}\left\{\frac{\partial h_i/\partial a_i}{h_i} \;\; S_i(1-S_i)\right\} \\ &= \left[\frac{\partial}{\partial q}\left\{\frac{\partial h_i/\partial a_i}{h_i}\right\} + \left(\frac{\partial h_i/\partial q}{h_i} - \frac{\partial h_j/\partial q}{h_j}\right)(1-2S_i)\frac{\partial h_i/\partial a_i}{h_i}\right] S_i(1-S_i)\end{aligned}$$

where in the last equality we used $\partial S_i/\partial q = [(\partial h_i/\partial q)/h_i - (\partial h_j/\partial q)/h_j]S_i(1-S_i)$. Using eq. (1), we have

$$\frac{\partial}{\partial q}\left\{\frac{\partial h_i/\partial a_i}{h_i}\right\} = 0$$

which gives

$$\frac{\partial^2 S_i}{\partial a_i \partial q} = \frac{\partial h_i/\partial a_i}{h_i}\left(\frac{\partial h_i/\partial q}{h_i} - \frac{\partial h_j/\partial q}{h_j}\right)(1-2S_i)S_i(1-S_i)$$

Sign of the above expression is determined by the sign of the second and third parentheses. Note that

$$\frac{\partial h_1/\partial q}{h_1} = \frac{1}{q} > 0 \quad \text{and} \quad \frac{\partial h_2/\partial q}{h_2} = 0$$

and from lemma 1 we know that $1-2S_i < 0$ when $q > 1/\gamma$ and $1-2S_i > 0$ when $q < 1/\gamma$.

This implies that when $\mu > 1/\gamma$, $\frac{\partial^2 S_1}{\partial a_1 \partial q}<0$ and $\frac{\partial^2 S_2}{\partial a_2 \partial q}<0$ and when $q < 1/\gamma$, $\frac{\partial^2 S_1}{\partial a_1 \partial q}>0$ and $\frac{\partial^2 S_2}{\partial a_2 \partial q}>0$. □

Proof of proposition 2

Proof. Part I Using the matching probabilities at the equilibrium of the advertising subgame in eq. (4) (for a given q) and taking the derivative of the equilibrium expression for V with respect to γ yields

$$\frac{\partial S_1(a_1^*, a_2^*; q)}{\partial \gamma} = \frac{q}{(1+\gamma q)^2} > 0 \quad \text{and} \quad \frac{\partial S_2(a_1^*, a_2^*; q)}{\partial \gamma} = \frac{-q}{(1+\gamma q)^2} < 0.$$

Part II Since firms' advertising levels are proportional in the equilibrium of the advertising game, they do not have any effect on the consumer's chance of matching with the preferred firm. As a result, consumer's expected benefit V is only a function of platform's quality. Computing the effect of a change in q at the equilibrium of the advertising subgame we get

$$\frac{\partial S_1}{\partial q} = \frac{\gamma}{(1+q\gamma)^2} > 0 \quad \text{and} \quad \frac{\partial S_2}{\partial q} = \frac{-\gamma}{(1+q\gamma)^2} > 0.$$

□

Proof of proposition 3

Proof. Obtaining q^* amounts to directly solving eq. (10). using eq. (3) and eq. (7) we have

$$\frac{A'}{A} = \frac{1-\gamma q}{\gamma q^2 + q} \quad \text{and,} \quad \frac{\hat{V}'}{\hat{V}} = \frac{\gamma(2\mu - 1)}{(\gamma q + 1)(\mu(\gamma q - 1) + 1)}$$

After Simplification, the objective then is to solve the following

$$\frac{\gamma\mu}{-\mu + \gamma\mu q + 1} - \frac{3\gamma}{\gamma q + 1} + \frac{1}{q} = 0$$

This amounts to solving a quadratic equation which delivers the results.

$$q^* = \frac{1}{\gamma} \underbrace{\frac{\sqrt{3\mu^2 - 3\mu + 1} + 2\mu - 1}{\mu}}_{\phi(\mu)} \tag{12}$$

It is also easy to verify that $\phi(\mu = 1/2) = 1$.

To see that $\phi(\cdot)$ is an increasing function taking derivative of the $\phi(\mu)$ expression in eq. (12) with respect to μ we get

$$\frac{d\phi}{d\mu} = \frac{3\mu + 2\sqrt{3(\mu-1)\mu + 1} - 2}{2\mu^2\sqrt{3(\mu-1)\mu + 1}} > 0$$

The square root term is always positive and reaches a minimum of $1/2$ at $\mu = 1/2$ and obtains 1 at $\mu = 0$ and $\mu = 1$ which renders the numerator always positive. Since $0 \leq \mu \leq 1$, both numerator and denominator are always positive. □